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ABSTRACT

This report describes a program for improving students' oral and written communication in mathematics. The target population consisted of third and fourth grade classes in a suburban, middle class community located in a large Midwestern state. The current lack of communication was demonstrated by the results of class surveys, student interviews, and teacher made tests. Analysis of national probable cause data indicated that communication in mathematics, with the exception of signs and symbols, has been clearly neglected. Discussion with faculty revealed that there is also widespread concern on the local level. Student surveys showed that students have not been given many experiences in oral and written communication in mathematics. Review of current curricular focus in the area of mathematics shows that communication is emphasized much less than computation. A review of solution strategies suggested by knowledgeable others, combined with an analysis of the problem setting, resulted in the selection and implementation of several intervention strategies. These strategies included the use of student math journals, cooperative groups, real life problem solving, and an increased emphasis on mathematical vocabulary. Post intervention data indicated an increase in the students' oral and written mathematical communication skills. Both the third and the fourth grade classes involved showed some improvement in their abilities to communicate mathematically. While the success of the third grade class was limited to more lower level communication skills, such as explaining what their answer is, the fourth grade class improved in all areas. This includes some of the higher-level communication skills, such as explaining how they got their answers and why they solved a problem the way they did. These improvements are documented and analyzed in this report. (Contains 39 references.) (Author/ASK)

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COMMUNICATION IN MATHEMATICS

Ben Huggins
Tim Maiste

An Action Research Project Submitted to the Graduate Faculty of the

School of Education in Partial Fulfillment of the

Requirements for the Degree of Master of Arts in Teaching and Leadership

Saint Xavier University & IRI/Skylight

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Abstract

This report describes a program for improving students' oral and written communication in mathematics. The target population consisted of third and fourth grade classes in a suburban, middle class community, located in a large midwestern state. The current lack of communication was demonstrated by the results of class surveys, student interviews, and teacher made tests.

Analysis of national probable cause data indicated that communication in mathematics, with the exception of signs and symbols, ha been clearly neglected. Discussion with faculty revealed that there is also widespread concern on the local level. Student surveys showed that students have not been given many experiences in oral and written communication in mathematics. Review of current curricular focus in the area of mathematics shows that communication is emphasized much less than computation.

A review of solution strategies suggested by knowledgeable others, combined with an analysis of the problem setting, resulted the selection and implementation of several intervention strategies. These strategies included the use of student math journals, cooperative groups, real life problem solving, and an increased emphasis on mathematical vocabulary.

Post intervention data indicated an increas in the students' oral and written mathematical communication skills. Both the third and the fourth grade classes involved showed some improvement in their abilities to communicate mathematically. While the success of the third grade class was limited to more lower level communication skills, such as explaining what their answer is, the fourth grade class improved in all areas. This includes some of the higher-level communication skills such as explaining how they got their answers and why they solved a problem the way they did. These improvements are documented and analyzed in this report.

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CHAPTER 1

PROBLEM STATEMENT AND CONTEXT

General Statement of the Problem

Students in the targeted third and fourth grade classes lack meaningful communication skills in mathematics. These skills include talking and writing about mathematical concepts and processes. Evidence for the existence of this problem includes teacher observation and teacher assessment of student math journals.

Immediate Problem Context

The two targeted classrooms in this study are located at two separate schools. These schools are located in the same school district and in the same community. The fourth grade classroom is designated as Site A with the third grade classroom as Site B.

Site A

Site A is a K-5 building. In 1995 it moved to a new facility two blocks from the old site. It moved to its new site because of student overcrowding. The current building is the district's newest and is considered state of the art. Amenities include: shared spaces

between classrooms equipped with phones, bathrooms in every primary classroom, air conditioning, large classrooms, and ample storage space in the classrooms.

New construction of single family homes, surrounding the school, has resulted in continual growth of the student population. In 1995 the student population was 517 (1994-95 School Report Card). Currently the enrollment is 782 students, which makes it the largest elementary school in the district. Overcrowding has again become a problem. The average class size is 27 students, which is above the district and state averages (1996-1997 School Report Card).

Four point seven percent of the students come from low-income families. The population of Site A is 90.7% White, 3.7% Asian, 3.1% Hispanic, and 2.5% Black. The school has an overall transient rate of 8.2% annually (District records). Students have consistently scored above the state averages on the IGAP test (1996-1997 School Report Card).

The school district has set guidelines for the amount of time spent on the core subjects and specials. The allotted time for reading instruction is 450 minutes per week. language arts gets 400 minutes of attention per week. Two hundred and fifty minutes per week are for math, while 150 minutes each week are set aside for social studies and science. In regards to specials, physical education gets 100 minutes a week, art is allotted 45 minutes per week with music class getting 25 minutes per week.

The school employs 27 classroom teachers, 3 of whom are male, and 19 specialists including an inclusion facilitator responsible for 18 inclusion students. Support staff includes a secretary, two clerical aides, four custodians, two crossing guards, one nurse,

one health aide, 10 lunchroom workers, four reading aides, 12 inclusion aides, and two Library Learning Center (LLC) aides.

The school has a before and after school program with 54 children. Extracurricular activities are also offered such as: Girl and Boy Scouts, Hands on Science, a computer club, a drama club, intramural basketball, and a recess football league.

Site B

Site B is a K-5 building. The building was constructed in 1969 utilizing the open classroom concept. Many of the classrooms are arranged in a half circle around the LLC. These classrooms have only one permanent wall. The other three walls are movable and not entirely sound proof. These classrooms are smaller in size than the district's standards.

An addition was built onto the site during 1997. This addition includes two classrooms, a music and an art room, and a gymnasium. It has also added teacher storage space and two bathrooms.

The student population is all within walking distance with the exception of one busload of children who live in an apartment complex. The student enrollment is 484. The average class size is 21.2 students, which is slightly below the district average (1996-1997 School Report Card).

Three point five percent of the students come from low-income families. The population of Site B is 95.2% White, 2.1% Hispanic, 1.4% Black, and 1.2% Asian. The school has an overall transient rate of 11.0% annually (District Records). Students have consistently scored above the state averages on the IGAP test (1996-1997 School Report Card).

The school district has set guidelines for the amount of time spent on the core subjects and specials. The allotted time for reading instruction is 450 minutes per week. language arts gets 400 minutes of attention per week. Two hundred and fifty minutes per week are for math, while 150 minutes each week are set aside for social studies and science. In regards to specials, physical education gets 100 minutes a week, art is allotted 45 minutes per week with music class getting 25 minutes per week.

The school employs 17 classroom teachers, 1 of whom is male, and 10 specialists. Support staff includes a secretary, two clerical aides, three custodians, two crossing guards, one nurse, one health aide, eight lunchroom workers, two reading aides, and one LLC aide.

The school has a before and after school program with 22 children. Extracurricular activities are also offered such as: Girl and Boy Scouts, intramurals, and a computer club.

The Surrounding Community

The school district is located 30 miles west of a large metropolitan area. Its population of 54,753 resides in an 11.17 square mile area. The population is divided into 93% White, 4% Asian, 2% African American, and 1% Other. The average age of a member of the community is 35.6 years old. The median family income is $77,558. Low-income families make up 6.2% of the population.

The unemployment rate is well below the rate for the state and the surrounding county. The county is the largest single employer with 2,700 people on the payroll. The school district is the third largest employer, while a local private college is the fourth largest employer.

The median home value is $193,263. A majority of the homes (74.5%) are single family dwellings. The make up of the town has changed over the years. Outlying cornfields have been replaced by ambitious housing developments where the houses cost as much as $500,000. The downtown area contains mainly older architecture revealing its 1800's roots.

There is a large community center, library, and courthouse. Residents also enjoy a plethora of paths for biking and walking. The two Metra stations transport 3,200 commuters to Chicago every day. The community contains 40 places of worship and 14 public childcare centers.

The School District

The school district contains two high schools, four middle schools, and 13 grade schools servicing more than 13,000 students. The superintendent was given a salary of $140,000 in 1996. The average administrator salary is $71,769, while the state average is $70,183.

The average teacher salary is $48,867, while the state average is $42,429. The percentage of teachers with a bachelor's degree is 41.7% with 58.3% holding a master's degree or higher. In comparison, districts of similar size in Illinois report 48.6% of their teachers hold a master's degree or higher. The average teacher has 13.9 years of teaching experience. The teachers of the district are 97.3% White, 1.3% Black, 0.7% Asian, and 0.7% Hispanic. At the elementary level, the teacher to pupil ratio is 20.7:1.

The district spends $5,733 per pupil compared to a $6,158 average for the state. The student population is divided into 87.1% White, 5.0% Asian, 4.0% Black, 3.8%

Hispanic, 0.1% Native American. The attendance rate is 95.9%. The dropout rate is 1.9%. The truancy rate is 0.7%. The mobility rate is 1.5% (District records).

National Context of the Problem

Students must learn to communicate mathematically. With the exception of signs and symbols, communication in mathematics has been clearly neglected. (National Council of Teachers of Mathematics as cited in Wadlington, Bitner, Partridge, Austin, 1992) As a result, the NCTM has developed a set of standards for the teaching of mathematics that focuses on this shortcoming in mathematics instruction. Standard 2, of these widely accepted standards, states that

> the study of Mathematics should include numerous opportunities for communication so that students can relate physical materials, pictures, and diagrams to mathematical ideas, reflect on and clarify their thinking about mathematical ideas and situations, relate their everyday language to mathematical language and symbols, and realize that representing, discussing, reading, writing, and listening to mathematics are a vital part of learning and using mathematics" (NCTM 1989).

This NCTM standard stresses the need for students to communicate mathematical ideas to themselves and to others orally and through written language.

The abilities of describing, explaining and justifying are critical in mathematics "because they help children clarify their thinking and sharpen their understanding of concepts and procedures" (Catchcart, Pothier, & Vance, 1994).

Other experts in education have noted this problem. Science for All Americans: a Project 2061 Report states that effective teaching of Mathematics requires the

development of oral and written communication of math concepts (American Association for the Advancement of Science as cited in Greenes, 1992). Another study by four university professors states that some "students ... view mathematics only as a series of memorized rules and formulas" (Wadlington, Bitner, Partridge, Austin 1992). This student perspective clearly undermines true mathematical understanding. "The level or degree of understanding of a concept or idea is intimately related to the successful communication of that concept or idea. Understanding is enhanced by communication" (Greenes, Schulman, Spungin 1992). Activities that require communication allow students to construct, refine, and consolidate their understanding of mathematics (Phillips & Crespo, 1995).

Teachers need to integrate communication into all aspects of their mathematics instruction. The Carnegie Council on Adolescent Development, in its study Turning Points: Preparing American Youth for the Twenty-first Century, encourages teachers "to promote a spirit of inquiry and to stimulate students to think about and communicate ideas" (The Carnegie Council on Adolescent Development as cited in Greenes, 1992).

CHAPTER 2

PROBLEM DOCUMENTATION

Problem Evidence

In order to document the extent of students' inability to communicate mathematically, student interviews were developed and administered in February 1999 (Appendix A). A pretest was also developed and administered by the researchers (Appendix B and C) to further document the problem and to provide the researchers with baseline data in order to determine the effectiveness of the interventions. In addition, student surveys were given (Appendix D). Teacher observations also provided valuable evidence documenting this problem.

Site A

Interviews were conducted at Site A. The students being interviewed were asked to arrange four numbered cards so that they would form a number as close to 5,000 as possible. The cards had the numbers five, nine, two and six on them. After watching the students complete the task, the interviewer asked questions like, "What number did you make?" "Why did you put that number there?" and "How do you know that this number is the closest to 5,000?" The interviewer was encouraged to deviate from the prepared questions into other areas that would delve further into the child's ability to

communicate. While the children were fairly strong at getting the answer, the interviews revealed an overall inability to successfully communicate how they got their answer and why they did what they did. After arranging the numbers successfully, one student who was asked to explain why he arranged the numbers in the order h did could only state, "This was the only way that would work." Another answered the same question with, "No other arrangements would get you that close." Neither one of the aforementioned students, along with any of their peers, mentioned place value or comparing the effect of one number to another in a particular column. Even after probing deeper, the interviewer was unable to get much more out of the majority of students that related to how they arrived at their answer and why they chose to do what they did. By contrast, almost all students were able to get the correct answer.

The pretest for Site A, given in late January, was titled "Dream Christmas" (Appendix B). The children were given a catalog showing items available for purchase at "The Twenty Dollar Store." They were assigned to purchase Christmas presents for family and friends at this unique store that only accepts twenty-dollar bills as payment. Then students were asked to calculate how many twenty-dollar bills they would need to give the cashier in order to cover the cost of their purchases. Finally, students were asked to calculate the change that they would receive. Students would be graded on how well they explained what the answers were, how they arrived at those answers, and why they did what they did to get their answer. They were required to explain, in detail, everything they did to complete the tasks mentioned above.

The researchers developed a rubric for this written pretest that would assess students' abilities to communicate what the answer was, how they got that answer, and

why they chose to do what they did. The rubric is based on the rubric used by the Illinois Goals Assessment Program for scoring their mathematical written communication problems. Their rubric was adjusted to delve deeper into what makes good mathematical communication and in order to make it more specific to this situation. The children would earn points based on their abilities to communicate the what, how and why in the three following tasks: 1) Finding the total cost, 2) Finding how many twenty dollar bills were needed to pay for the items, and 3) Finding how much change they would receive back. Students earned 3 points for communicating what their answer was in all three tasks, 2 points for communicating what their answer was in two of the tasks, 1 point for communicating what their answer was in one of the tasks and 0 points if they communicated what they did in none of the areas. The overall perfect score was 9, if they communicated in all areas successfully, while the minimum was 0 for not communicating their results in any of the areas mentioned. It should be noted that whether or not the answer given was correct was of no factor in this research. Only their effort to communicate what they did was assessed, even if the wrong answers were given.

This pretest was completed in January 1999. The results of the pretest are outlined in Figure 1, Figure 2, Figure 3, and Figure 4.

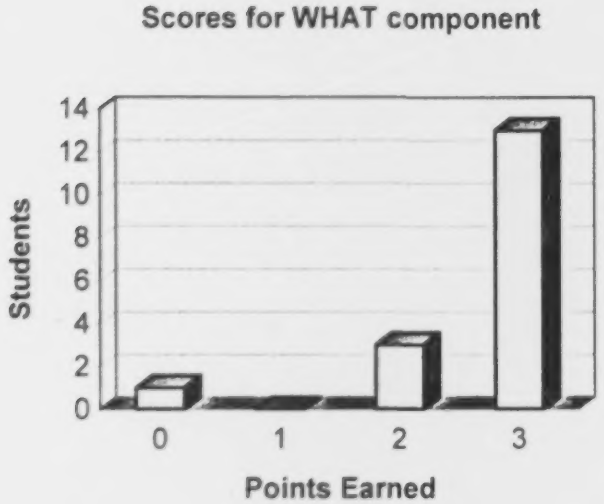

Figure 1. Student scores on pretest for the "what" component at Site A.

Of the seventeen students tested, one earned a 0 for the "what" component of the pretest. None of them scored 1, while three of them earned a score of 2. A full thirteen out of seventeen scored a perfect score of 3 on this portion of the test. Seventy-six percent of the students in this class were able to successfully communicate what their answers were to each part of the pretest, while only 6% scored less than 2. This clearly shows that communicating the answer is not a problem at all for the students at Site A.

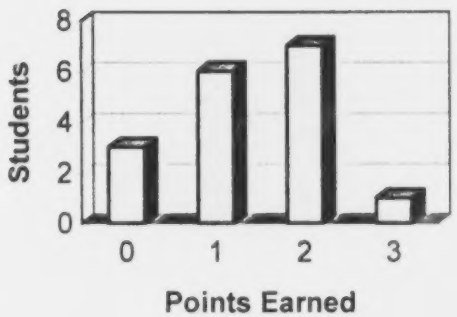

Figure 2. Student scores on pretest for the "how" component at Site A.

Of the seventeen students tested, three earned a 0 for the "how" component of the pretest. Six of them scored 1 while seven of them earned a score of 2. Only one child out of seventeen scored a perfect score of 3 on this portion of the test. Fifty-three percent of the students in this class only communicated how they arrived at their answers on one or fewer tasks. A similar sized group of 47% were able to successfully communicate how they arrived at their answers on two or three of the tasks. Only 6% received a perfect score. The majority of the children clearly fell into the category of successful communication on one or two tasks. In general, it is shown that half the class is doing pretty well, while the other half is struggling with explaining the how aspect of mathematical communication. There is certainly room for growth in 94% of the students.

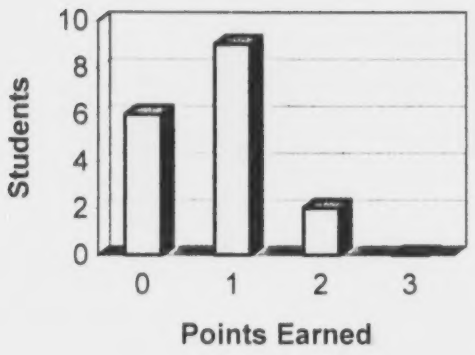

Figure 3. Student scores on pretest for the "why" component at Site A.

Of the seventeen students tested, six earned a 0 for the "why" component of the pretest. Nine of them scored 1 while two of them earned a score of 2. No children out of seventeen scored a perfect score of 3 on this portion of the test. A full 88% of the students in this class were not able to successfully communicate why they chose to do what they did, scoring 1 or lower. Only 12% were able to successfully communicate why they did what they did, scoring 2 or 3. The majority of the children clearly were unable to successfully communicate. There is great room for growth in the area of communicating why students chose to do what they did when solving a problem.

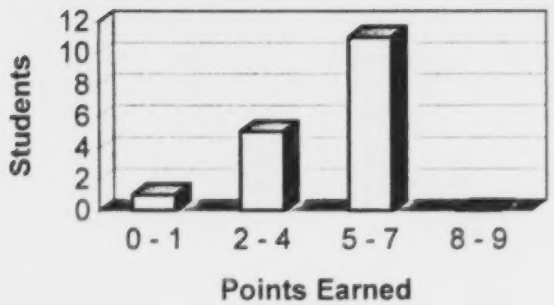

Figure 4. Student overall scores on pretest at Site A.

Of the seventeen students tested, none of them were able to score an 8 or a 9 overall on the pretest. Eleven of them scored in the 5 to 7 range, while five of them earned a score in the 2 to 4 range. Only one child out of seventeen scored in the 0 to 1 range. Not a single child was able to communicate in all areas successfully. A majority, 65%, were able to do well enough to score in the 5 to 7 range, showing some proficiency but also room for growth. This scoring range can be described as "adequate." Thirty-five percent of students scored "below average," earning a score of 4 or less. These students certainly struggle with communicating mathematically.

A survey was another tool that was used to collect data about student perception of their abilities as math students. The survey also asked students to rate how often they are asked to explain how they found their answers to math problems out loud or in writing.

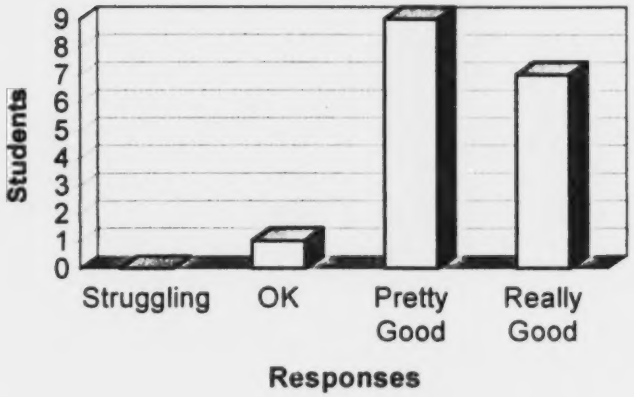

Figure 5. Student responses from Site A to the question, "Which of these would best describe you as a math student?"

When given a survey, the students' responses at Site A demonstrated a confidence in their math abilities. No student described him or herself as a struggling math student, and only one said he was just OK at math. However, nine students said that they were pretty good at math, while seven more responded that they were really good at math. Thus, 94% of the children have a positive view of their math abilities compared to only 6% who recognize that math may be a weakness for them.

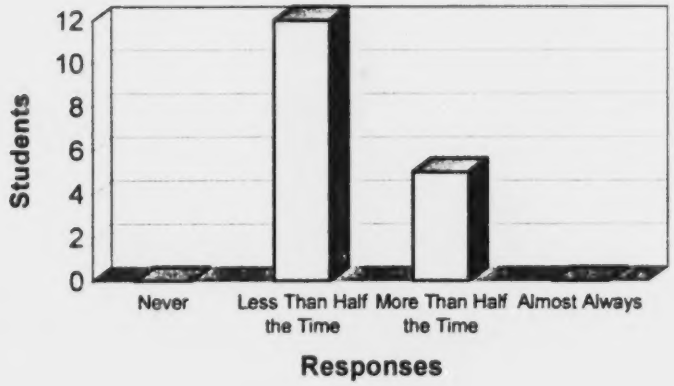

Figure 6. Student responses from Site A to the question, "How often are you asked to explain in writing the way you found an answer to a math problem?"

There were not any students who felt they were never asked to explain in writing their method for answering a math problem. Twelve students felt that this was asked of them less than half the time. Another five of the children felt that they had been asked more than half of the time to explain in writing how they solved a problem. No students felt that they were always asked to describe in writing how they solved a math problem. These data indicate that part of students' problems with mathematical communication can be traced to this lack of experience with communicating in written form. A full 71% of students felt that they were asked to communicate their methods and thought processes less than half the time. This indicates that certainly the majority of the time, teachers at Site A only require that their students communicate their answers. This corresponds to the what section of the pretest data in which most students at Site A were successful at communicating the what portion of their answer.

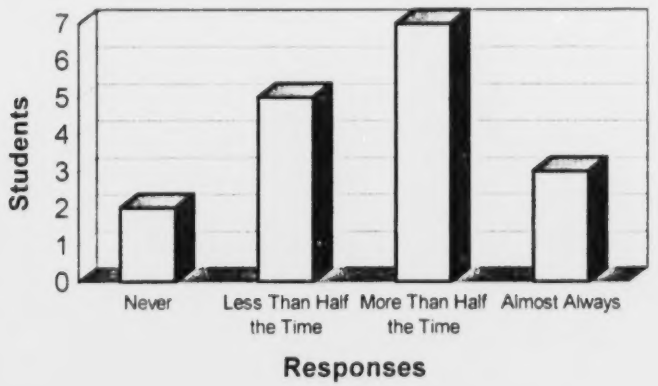

Figure 7. Student responses from Site A to the question, "How often are you asked to explain aloud the way you found an answer to a math problem?"

In response to this question of the survey, two students responded that they were never asked to explain aloud the way they found an answer to a problem. Five students thought that they were asked to explain aloud their methods for answering a problem less

than half the time. Seven students responded that they were asked to explain aloud the way they solved a math problem more than half the time. Three indicated that they were always asked to explain their answers and methods aloud. While the numbers for this question definitely reflect a stronger emphasis that educators put on oral communication over written communication, they still leave 41% of students feeling like they are asked to communicate in this way less than half the time. This includes explaining their methods to the class, a partner, a small group or just the teacher. A meager 18% feel they are asked to explain their answers aloud almost all the time. These data further indicate the students' lack of experience in communicating mathematically, specifically in oral form.

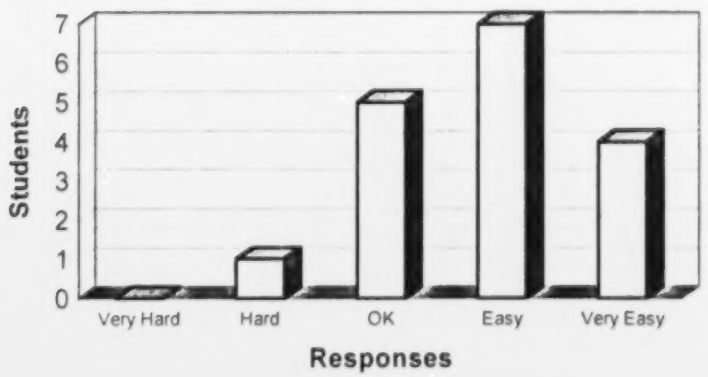

Figure 8. Student responses from Site A to the question, "How easy is it for you to explain how you solved a math problem?"

Four out of seventeen students considered it to be very easy for them to explain how they solved a math problem. Out of seventeen, seven responded that it was easy to explain how they solved a math problem. Five out of seventeen students felt they were just ok at such explanations. Only one felt that such explanations were hard, while no one thought that explaining how they solved a math problem was very hard. Surprisingly, this graph

shows that 65% of students find explaining how they answered a math problem was at least easy. Thirty-five percent lacked confidence in their ability to communicate their solutions.

Site B

Interviews at Site B required students to arrange four numbered cards into a number as close to 5000 as possible. The four numbers were five, nine, two and six. Most of the students were successful in the task. When asked to explain their answer the students were not as successful. One student was asked why he made that number. His response was, "If it's over then you want to put the little numbers closer to it. If it's lower then you put the higher numbers first so it'll be a higher number." Another student answered, "The five is in the thousands. The two is the lowest number. Six and nine are higher numbers." The students were able to get some of their points across minimally throughout the sections of the interview. However, none of the students explained themselves in a manner that was easy to understand.

At one point in the interview the number five was exchanged for the number four and the student was asked to make a new number that was closest to 5000. Most students were again able to complete the task correctly, but had difficulty justifying their decisions. "I made 4,962," one student explained. "Why?" "Well, just like something makes 100 you'd take 96 because it's closest." While these explanations made some sense when the students pointed at the numbers they were referring to, the communication was not as effective as it could have been.

The pretest for Site B consisted of one multiple-part open-ended question involving earning money by scheduling tasks for one week. The main purpose for this

pretest was to evaluate how well the students communicated their answers. The rubric developed by the researchers has three parts (Appendix D). These three parts are what, how and why. Students were evaluated on if they told what the answer was in their written explanation. Students were evaluated on if they explained how they came up with their answers. Students were also evaluated on if they explained why they did their work the way they did. Each portion of the rubric earned a score from 0 to 3. A score of 3 was given if a student told all parts of what the answer was. A score of 2 was given if a student told most parts of what the answer was, and a score of 1 was given if the student gave less than half of what the answer was in his written explanation. A score of 0 was given if the student didn't tell any part of what the answer was in their written explanation. The same standards are applied to the other two parts of the evaluation.

This pretest was completed in January 1999. The results of this pretest are outlined in Figure 9, Figure 10, Figure 11, and Figure 12.

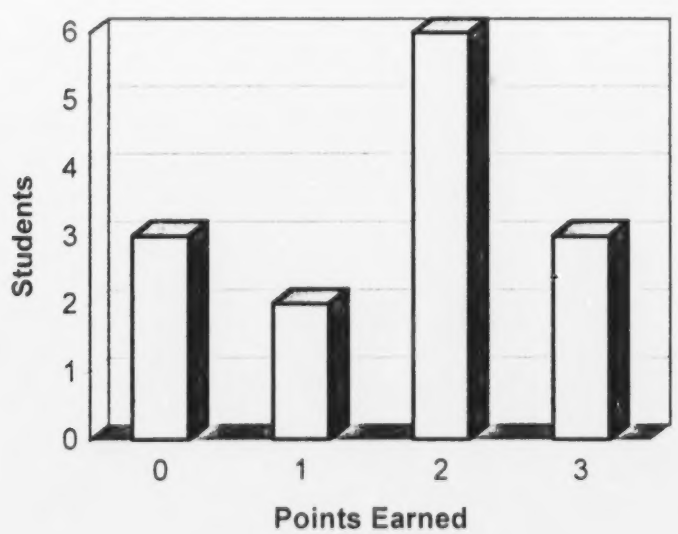

Figure 9. Student scores on the "what" portion of the pretest given at Site B.

Of the fourteen students tested, three earned a score of 0 on the "what" portion of the pretest. Two students received a score of 1, six received a score of 2, and three received a score of 3. Sixty-four percent of the students scored either a 2 or a 3 on this portion of the rubric. Most of the students told most or all of what the answer was in their written response. This demonstrates that a majority of the students in the third grade class were able to tell specifically what the answer was for the problem posed or at least most of what the answer was.

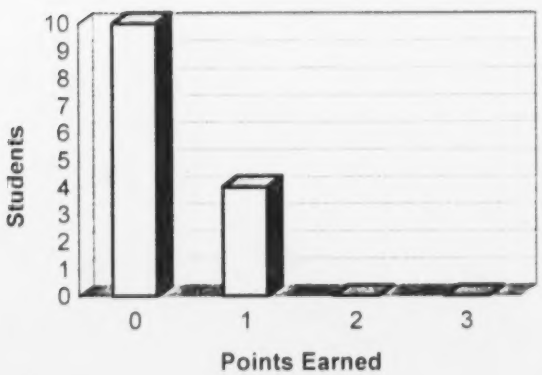

Figure 10. Student scores for the "how" portion of the pretest given at Site B.

Of the fourteen students tested, ten received a score of 0 on the "how" portion of the pretest. These students did not offer any explanation of how they arrived at their answers to the question posed. Four of the students earned a score of 1. These students explained how they found the answer to less than half of the question. None of the students scored a 3 or a 4 on this portion of the pretest. This shows that all of the students tested needed to improve in their ability to communicate how they solved a

problem. Most of the students were successful in telling what the answer was, but none of them explained fully how they arrived at their answers.

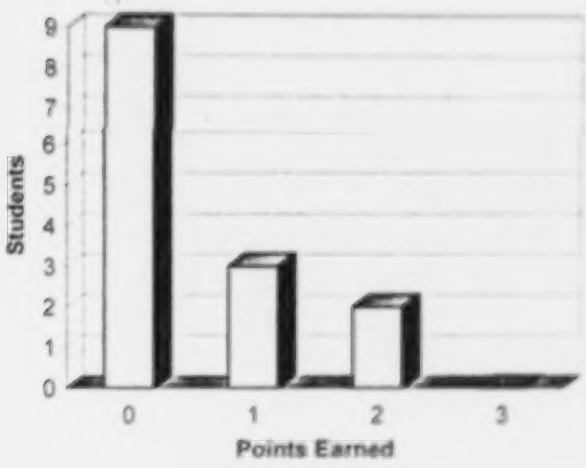

Figure 11. Student scores for the "why" portion of the pretest given at Site B.

Of the fourteen students tested, nine received a score of 0 on the "why" portion of the pretest. Three students received a score of 1. Two students received a score of 2, and zero students received a score of 3 on the "why" portion of the pretest. Sixty-four percent of the students had no explanation at all about why they did the things they did in their problem. Only 22% of the students gave minimal explanation of why they did the things they did, and 14% of the students explained more than half of why they did things the way they did them. Eighty-six percent of the students, a large majority, told little or nothing about why they did their problem in the manner that they did it.

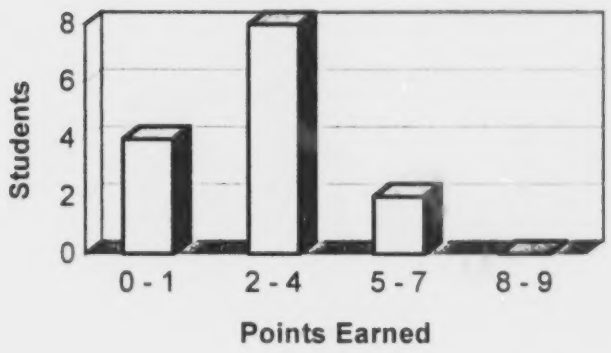

Figure 12. Student overall scores on pretest at Site B.

Four students received an overall score of 0 or 1 on the pretest. Eight students received a score of 2, 3, or 4, and two students scored between 5 and 7. Eighty-six percent of the students scored less than 5 points on the pretest, while 14% scored 5 or higher. A large majority of the students need to improve their overall skills in communicating in mathematics.

A survey was another tool that was used to collect data about student perception of their abilities as math students. The survey also asked students to rate how often they are asked to explain how they found their answers to math problems out loud or in writing.

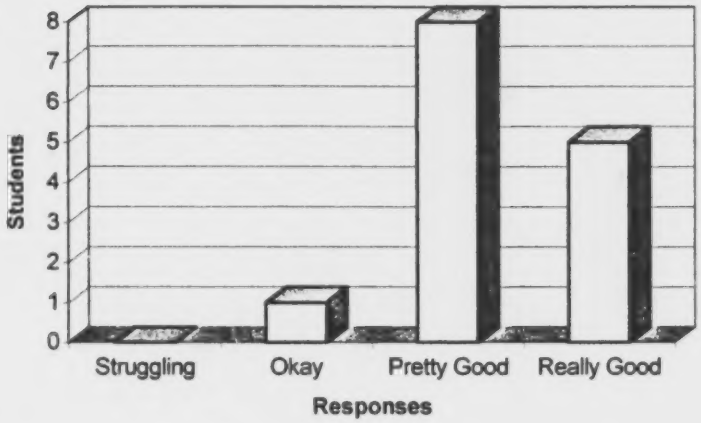

Figure 13. Student responses to the question, "Which of these would best describe you as a math student?" from Site B.

At Site B no students responded that they were struggling math students. One student answered that he was an okay math student. Eight students felt that they were pretty good math students, and five responded that they were really good math students. Ninety-three percent of the students surveyed said that they were either pretty good or really good as math students. This shows that most of the students at Site B felt confident in their abilities as math students.

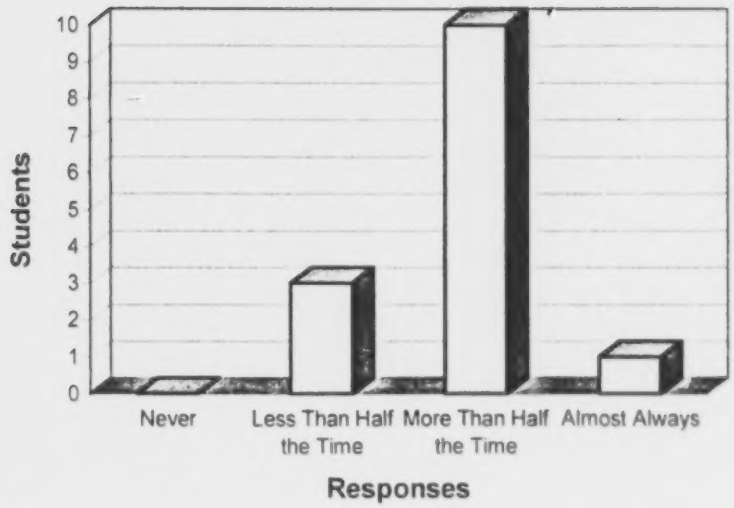

Figure 14. Student responses to the question, "How often are you asked to explain in writing the way you found an answer to a math problem?" from Site B.

At Site B no students responded that they were never asked to explain in writing how they solved a math problem. Three students answered that they were asked less than half the time to explain in writing how they solved a math problem. Ten students felt that they were asked to explain in writing how they solved a math problem more than half the time, and one responded that they were almost always asked to explain in writing how they solved a math problem. A large majority of the students, 79%, responded that they were asked to write about math most or almost all of the time. This shows that the students' perception was that they were writing a significant portion of the time during math.

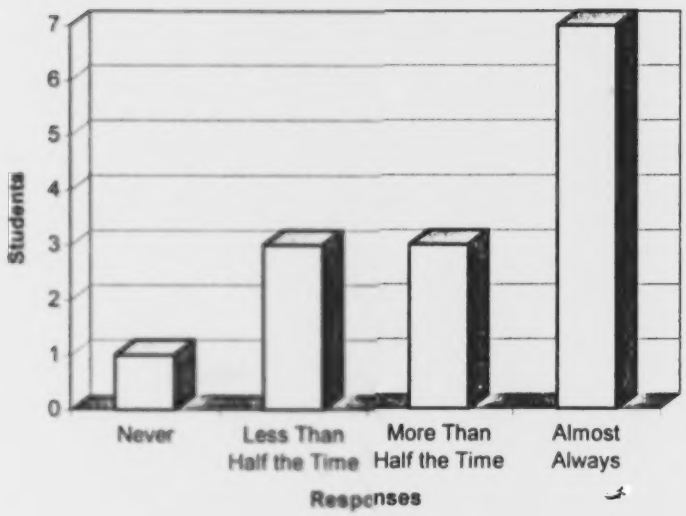

Figure 15. Student responses to the question, "How often are you asked to explain aloud the way you found an answer to a math problem?" from Site B.

At Site B one student responded that he was never asked to explain aloud how he solved a math problem. Three students answered that they were asked less than half the time to explain aloud how they solved a math problem. Three students felt that they were asked to explain aloud how they solved a math problem more than half the time, and seven responded that they were almost always asked to explain aloud how they solved a math problem. Half of the students responded that they were asked to explain how they solved math problems out loud almost always.

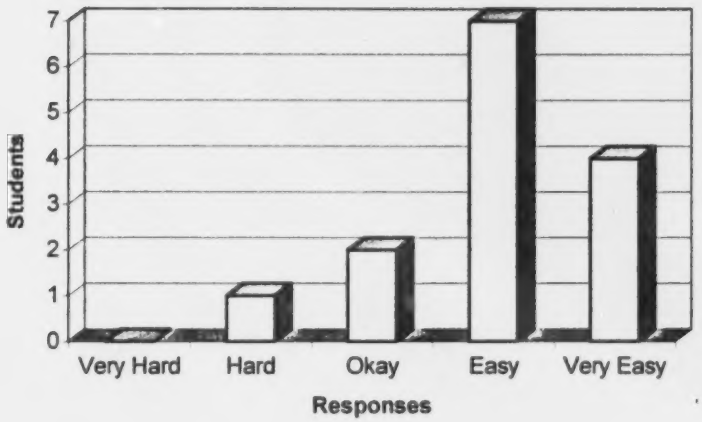

Figure 16. Student responses to the question, "How easy is it for you to explain how you solved a math problem?" from Site B.

At Site B no students responded that explaining a math problem was very hard for them. One student responded that explaining math problems was hard for him. Seven students felt that it was easy for them to explain how they solved a math problem, and four students said that it was very easy to explain how they solved a math problem. A majority of the students, 79%, responded that they felt that it was easy or very easy for them to explain how they solved math problems. The students' perceptions did not match their performance on the pretest. None of the students were able to communicate how they solved the problem posed on the pretest. In fact only four of the fourteen students even attempted to explain how they solved the problem.

Probable Causes

The literature states that mathematics students are rarely given opportunities to write what they are thinking (LeGere, 1991). Students are not asked to practice mathematical communication in the classroom very often. One of the reasons for this is that many teachers rely on the mathematics textbooks to provide the content and structure of their lessons. Many textbooks focus on isolated paper and pencil drill, with little or no mention of communication (Dossey et al. 1988, as cited in Parker, 1993). This keeps students from making sense of mathematics. They do not make the connection between math and their world (Countryman, 1992).

Traditionally, activities, including mathematics communication, have dealt almost exclusively with signs and symbols. Other kinds of communication have been used infrequently. Communicating mathematics through writing has been particularly neglected (Wadlington et al. 1992).

Donald Graves is a researcher who has found that there is a lack of writing instruction in the language curriculum. In some of the surveys that he conducted he found that in the teaching colleges, 169 courses were offered in the teaching of reading while only two courses were offered to help teach writing (Graves, 1987). He also found a huge difference in the amount of money being spent for research by the department of education. For every dollar spent on the study of writing, three thousand were spent on the improvement of reading.

As a result of the lack of training provided for new teachers, many teachers do not implement writing in their mathematics classroom. In a study by Davidson and Pearce (1988), data on the amounts, kinds and uses of writing in mathematics classes were

collected. The study found that writing was seldom used in instruction. Writing happened in the mathematics classroom on an average of one activity every two weeks. Teachers are teaching mathematics in the same ways that they were taught mathematics (Parker, 1993).

Writing has not been used across the curriculum in the past. One of the reasons reported by teachers is that the school day is already packed with the current curricular demands, and the time for writing is not available. Writing also consumes large amounts of the teacher's time. Reading and commenting on each student's writing takes up a lot of time. Another reason writing has not been used is that many teachers feel pressure to have their students do well on the standardized tests that are used by principals and districts to determine which teachers are successful in their teaching methods. Fortunately, in the state of Illinois, the ISAT test, which is administered in several grade levels from 3rd to 12th grades, has begun to incorporate written communication in its mathematics testing. This will encourage teachers to make this a focus in their classrooms.

Students have also not had opportunities to communicate verbally in mathematics (Adams & Cooke, 1998). Often, teachers ask students to solve a problem, listen to an incorrect response, then move on to another child until they hear what they were looking for. In doing this, opportunities for conversation, discussion, and challenging ones own guesses are missed. Mathematical concepts are not developed in the absence of mathematical language (Lilburn & Rawson, 1994). Without these opportunities students' are not able to learn the language of mathematics. Because they have not learned this language they are not effective in communicating mathematically (Krussel, 1998).

Too often children learn to compute without understanding why the computation procedures make sense. Missing from instruction has been attention to children's explaining the procedures they use, justifying their reasoning, judging the reasonableness of their solutions, and reflecting on their thinking. For many students the quick right answer has become more valued than the thinking that led to that answer. Mathematics is seen as a subject that is communicated through the manipulation of symbols in orderly ways, not as one that uses words to express ideas (Burns, 1995).

Another reason that students are poor at communicating mathematically is the focus of the curriculum. The mathematics curricula in many districts are centered on the memory and mastery of facts and formulas rather than seeing relationships and communicating those relationships to others (Parker, 1993). Authentic application of math concepts is missing from current instruction.

The literature also indicates other possible causes for students' inability to communicate effectively in math. Current textbooks and curriculum are often outdated and focus on a set approach to mathematics. The students see no relevance to their own lives in the content. The research shows that in order to be engaged in writing, students must have a sense of ownership in the task. The work must become their own (Schoenfeld, 1989). Students need to develop their own way of interpreting an idea, relating it to their own lives, and connecting it to prior knowledge (Daniels, Hyde, & Zeleman, 1993). Unless they work through it in their own way, those connections will not be made.

Current forms of assessment also contribute to ineffective written mathematical communication. Even those teachers who employ writing techniques in their math

classroom still often revert back to traditional methods for assessment (Burke, 1997). Those traditional methods place an emphasis on inert knowledge, calculations, and formulas (Brown, 1989). This sends a confusing message to the student as to what is really important (Burke, 1997). If communicating mathematical knowledge is so important, it would not be ignored during assessment.

CHAPTER 3

THE SOLUTION STRATEGY

Literature Review

There are several solutions to our problems. Many sources validate the belief that the more you do something, the better you will become at it. Pavlov's experiment with the dogs illustrates this point nicely. When the dog heard the bell he knew food was coming and he would salivate. The same needs to be true for children. "Everyone get out your math journals!" should elicit a similar response in our students (Huggins & Maiste, 1999).

"Writing is a vehicle for clear and logical communication" (Myers, 1984). In order for students to deepen their understanding of mathematical concepts students need to express their ideas to others. One way to do this is through writing. "The productive use of language is a skill that all students should practice in all disciplines; reading, writing, and speaking belong in every classroom, even math classrooms" (Countryman, 1992, p.2).

Writing does a number of things for the students and for the teacher. "Writing about their thinking can help children gain clarity and can reveal their level of

understanding" (Parker, 1993, p.156). Writing about a mathematical concept allows students to discover their level of understanding.

It will also allow the teacher to see what the students understand. According to Lilburn and Rawson (1994) "Language gives us valuable information about where the children are at, as well as insights into their understanding." Hidden misconceptions will be exposed through writing that would otherwise go undetected (Stix, 1994).

Through writing students are forced to think metacognitively. This also allows students to clarify issues for themselves. Writing engages the students in the mathematical concepts being taught. It allows the students to transfer their thoughts into written form. "Thus, writing encourages processing" (BeMiller, 1987). Writing allows students to explore the connections between what they know and what they are learning. Writing also makes students think about the subject in a very focussed way. "Writing demands an internal monologue on the ideas under consideration" (BeMiller, 1987). A residual effect of writing is that it can clear the mind of distractions and barriers to learning. You can interact with your thoughts when you put them on paper. Taking the concept out of your head and putting it on paper allows you to see it. This allows you to make connections with other ideas and concepts.

Another benefit of using writing with students in the mathematics classroom is that math is no longer viewed as a collection of right answers. "Writing mathematics can free students of the assumption that math is just a collection of right answers to questions posed by someone else" (Countryman, 1992, p. 11).

In a study by Stix (1996), teachers in a mathematics for elementary teachers class were given journal assignments to complete for the class. The teachers involved reported

that their math anxiety decreased and their self-confidence increased as a result of journal assignments.

In another study conducted by Schubert (1987) one class of fourth graders used math journals during their unit on fractions. The second class did not. The following year when the students were mixed together, the students who had used journals the previous year scored significantly higher than the rest of the class. The average score for the journal group was 94%. The rest of the students had an average of 81%. The teacher concluded that, "what you write about, you understand and remember" (Schubert, 1987).

The fifth grade teacher observed that students who had used journals seemed less anxious than the students who had not used a journal in fourth grade. BeMiller (1987) also suggests that, among other benefits, informal writing also relieves math anxiety.

Free writing is a short period of writing in which students are to write about anything that is related to mathematics. They may choose to ask questions. They may write about what they learned about the previous day, or they may simply write about what's on their minds about math. The only guidelines are that students should write whatever comes to mind, and it should be a fixed, short amount of time (Countryman, 1992).

Learning logs are another form of writing in the mathematics classroom (Countryman, 1992). Students are given 10-15 minutes at the end of each class period to record what they've learned and to reflect on their understanding. These "serve as personal records of the experience of doing mathematics" (Countryman, 1992, p.20). These learning logs should be used on a daily basis (McIntosh, 1991).

A math journal is a good place to keep many forms of written communications about math. According to Countryman (1992) there are many purposes of journals. Journals can increase student confidence and participation. They allow teachers and students to monitor progress and record growth. Journals enhance teacher/student communication and can replace quizzes and tests as a means of assessment. Journals are similar to learning logs. The main difference is that journals are less formal and enhance communication more effectively (McIntosh, 1991).

Another form of communication in mathematics is speaking. Math talk provides many of the same benefits as writing. Oral communication allows other students to hear how their classmates are making sense of the concepts being learned. Lilburn and Rawson (1994) feel that talking about math should precede any written component of a math class. "Only after children have had many opportunities to talk about and work out situations orally in groups, should we expect them to write down, or read examples in symbolic form" (p. 4).

Math talk needs to take place in many different settings (Adams & Cooke, 1998). Students should talk about math in small groups. They should also explain their math reasoning to the entire class. One on one math talk should also occur frequently. These interactions should be focused more on the construction of meaning, than on the delivery of facts. The question to the student needs to be, "How did you get that answer?" rather that "What answer did you get?" The "How?" is much more important than the "What?" Asking students to explain how they found an answer requires students to think about the language of mathematics.

Students benefit from the use of oral language. It is a powerful tool that helps children make sense of ideas and relationships. The children should be talking about and explaining their ideas.

Hearing students talk about what they understand allows the teacher to fill in the pieces that are missing from a child's understanding. Thus, the concept can be grasped more fully. "Active discussions are a valuable vehicle for learning; they are, for teachers of the young child, the best window to the young mind. The focus ... is on the development of children's thinking not on the production of sterile answers" (National Council of Teachers of Mathematics, 1989, pp. 1-15). In order to increase students' ability to speak the language of mathematics, students need to be engaged in communication daily (Krussel, 1998; & National Council of Teachers of Mathematics, 1989, pp. 1-15).

Another solution for this problem is cooperative learning (Bellanca & Fogarty, 1991). Cooperative learning takes place in small groups. In these groups students are placed heterogeneously. The size of these groups varies from two to five or six. They work collaboratively to complete academic tasks.

The structure of these groups requires that all students participate. Members of the group rely on each other to complete tasks successfully. As a result of this, student learning is enhanced. Another result is that students are engaged in communicating their understanding of the concepts involved to other group members (Artzt, 1994). This group discussion allows students to talk, listen, explain, and think with others (Davidson & Pearce, 1990).

In cooperative groups students with a range of skills are able to help each other and be helped. The low achieving student reaps the benefits of a high achiever's brainpower, while the high achiever deepens his knowledge through the sharing and teaching of the skill or concept to others. Cooperative learning can be used to practice skills, investigate ideas, discover concepts and collect data. It also works well with technology instruction, peer tutoring and strategies such as brainstorming and reviewing. Also, research shows an improvement in students' attitudes towards learning through the use of cooperative groups (Artzt, 1994).

The literature also suggests that problem solving is a solution for improving communication in mathematics. Greenes, Schulman, and Spungin (1992) conclude that

> to develop communication skills and simultaneously enhance understanding of mathematics, students must be presented with problems that provoke curiosity and stimulate the need to describe, to justify, to explain, and to create. Students must have the opportunity to share their thoughts with others, to brainstorm and wrestle with ideas, and to get feedback and to make revisions to the first drafts of their thinking. (p. 82)

They encourage teachers to take on the role of orchestrator of these opportunities.

Problem solving creates a situation in which students need to communicate mathematical ideas. Because "problems in all categories have multiple solutions or multiple methods of solution," they naturally promote "discussion, debate, discovery, and creativity. It is important to recognize that learning to communicate mathematically is best accomplished in contexts where there is an authentic need to communicate"

(National Council of Teachers of Mathematics as cited in Phillips & Crespo, 1995). Problem solving provides that context.

Then how do teachers go about teaching problem solving in the classroom in a way that enhances communication skills? Problem solving instruction in the classroom should focus on teaching an overall plan for solving problems (Suydam 1982). Previous studies by Robinson (1973) support this idea. He found that, "Good problem solvers used a more formal approach" instead of just solving a problem with no plan or strategy specifically in mind.

Hyde and Hyde (1991) lay out four distinct steps that should be used when solving any problem: understand, plan, carry out, and review. Understanding the problem challenges the students to take a close look at what is really happening in the problem. In a way, place themselves in the problem to see what is really going on. After they understand the problem, they should plan out a course of action for solving the problem. Then they can carry out that plan. Finally, they should review their work and see if their answer makes sense.

Rincon and Ryan (1978), based on their findings, establish that students need more than just experience solving problems. They need instruction in the strategies for solving problems in order to develop as a problem solver. They concluded, "Intensive practice in problem solving is not sufficient to increase problem solving ability without instruction in specific techniques" (p.48). As further evidence Schifter (1996) suggests that the mathematics classroom should be a problem-posing and problem-solving environment, that a greater emphasis should be placed on developing an approach to thinking and a lesser emphasis placed on memorizing algorithms.

Problem solving, when used in conjunction with cooperative groups, provides for group discussion of problems, fostering communication by giving children opportunities for talking, listening, explaining, and thinking with others (Davidson & Pearce). These discussions are greatly enhanced when the problems being posed draw from the experiences of the children. Sinner (1959) strongly suggests that teachers formulate problems that draw on the direct experience of children. Developing problems that do this is the easiest way to connect with students' prior knowledge (Heibert, 1984), thus encouraging communication, conversation, responses, ideas, and, of course, learning. Peterson and Barnes (1996) state that teachers should be encouraged to develop lessons that present mathematical concepts in a format connected to their own lives. Their studies also showed that when a mathematical concept is connected to their lives, that students and teachers found them to be more interesting and more important.

Project Objectives and Processes

As a result of increased instructional emphasis on writing and speaking in mathematics, during the period of January 1999 to May 1999, the targeted third and fourth grade students will increase their ability to write and speak mathematically, as measured by teacher observations and teacher assessments.

In order to accomplish the project objective, the following processes are necessary:

1. Children will work in cooperative groups to solve and discuss math problems.
2. Problem solving strategies will be directly taught using the Problem Solver book. These will include real life problems.
3. Students will journal on a regular basis in mathematics.

Project Action Plan

The plan begins with the collection of data. The teachers will gather data using one open ended math problem, one questionnaire and one interview. This will take place during January and February 1999.

For the first measurement the teacher will administer pretests designed to measure students' ability to communicate mathematically in written form. These will be open ended math problems that incorporate real life problems.

The second assessment will be the questionnaire. The students will complete a questionnaire before and after the interventions are administered that is designed to provide the teachers with information about the quantity of the students' experiences with mathematical communication. It also provides teachers with students' perceptions about how well they communicate mathematically.

The final assessment is an interview. Before and after implementation of the interventions the teacher will conduct an interview with a selected sample of students. This interview is designed to get a feel for how the students communicate mathematical knowledge and processes orally.

Upon completion of the assessments, the teachers will implement three main strategies to help students improve their abilities to communicate mathematically. The strategies are: journal writing, cooperative learning, and problem solving. These will be used from the period of February 1999, to May 1999.

Journal writing was chosen to allow the students to write more often about math. Cooperative learning was chosen to allow the students to interact in a group discussion using the language of math. Problem solving was chosen because it will provide

opportunities, in the cooperative groups, to talk, listen, explain, and think about mathematics.

Journal writing will take place in a number of places. Students may write in a notebook. Some journal writing may take place on loose-leaf paper. Journals will be used at least twice a week. Sometimes they will be used in conjunction with problem solving.

At times the assignments will be open-ended. For example: students may be asked to write everything they know about multiplication, or students may be asked to write about what the most valuable thing they learned in math was this week. Some journal assignments will be more directed. For example: students may be asked to explain how they found the answer to an open-ended problem, or students may have to justify their answer to a problem and prove why they believe it makes sense. Journals will allow students to write their thoughts, think about how they solved a problem, and practice their communication skills.

Cooperative learning will take place once a week. Students will be placed in groups of three to five. Groups will be mixed heterogeneously by math ability. These groups will be presented with a real life, open-ended problem to solve. These groups will discuss the problem in order to find a solution.

Once the group discovers an appropriate answer to the problem, each group member will be responsible for explaining the solution to the problem and the method they used to find the answer in their math journals. When the entire class completes the written portion of the problem solving activity, a few of the groups will share their answers and methods of finding the answer with the rest of the class.

Methods of Assessment

In order to assess the effects of the intervention, students will complete a questionnaire before and after the interventions are administered which is designed to provide us with information about how the student feels about communicating mathematically. In addition, the teacher will conduct an interview with a selected sample of students. This interview is designed to get a feel for how the students communicate mathematical knowledge and processes orally. A test in which students are required to explain their answers and methods in writing will be given to measure student's ability to communicate mathematically in written form. A rubric will be developed to assess the quality of the communication component of the test.

CHAPTER 4

PROJECT RESULTS

Historical Description of the Intervention

The objective of this project was to improve the students' ability to communicate mathematical concepts. Interventions included cooperative group work, the direct teaching of problem solving strategies used to solve real life problems, and the use of math journals.

Cooperative learning was used as a way to teach the children to communicate verbally about mathematical concepts. The problem solving strategies were taught while the children were in cooperative groups. This allowed the students to work together and led them to use mathematical terms and descriptions when solving problems.

The students worked in these cooperative groups on a weekly basis beginning the last week of February. They worked on a "Problem of the Week" during these sessions. Most classes would begin with the direct teaching of a problem solving strategy. Once the strategy was taught and understood, the groups would be given a similar problem to solve. Once each group had come to a suitable solution to the problem a few groups were given the opportunity to share their results with the class. Most problem solving sessions were concluded with a journal writing time. This time was often used to reflect on the

process that was involved in solving the problem. Students were also instructed to explain what struggles they experienced in solving the problem.

Every four weeks the groups received a problem without any direct instruction. The groups then had to solve the problem using one or more of the problem solving strategies that they had learned in the previous weeks. Several of the problems used in this manner can be found in Appendix E.

Cooperative group skills were also taught at this time. One of the skills taught was sharing ideas. The importance of every group member participating in and contributing to the discussions was stressed throughout the intervention period. This was done in order to get the students talking about math ideas.

Journals were employed two to three times a week, depending on the schedule of that week. At times the journal assignments were open-ended. On one occasion the students were told to write everything they knew about multiplication. On three or four occasions students were asked to write about what the most valuable thing they learned in math was that week.

Some journal assignments were more directed. In several journal entries students had to justify their answers to different problems and prove why they believed their answers made sense. For some journal entries the students were required to explain how a mathematical process was done. For example: students were asked to explain to someone how to make a graph using some tallied data. Another time, students were told to write a detailed description of the concept taught that day for a classmate who was absent so that the absent child would be able to learn what he missed in class that day.

At times the students were allowed to write whatever they wanted about math. They could write about something they were thinking about, something they discovered in math that week, something they were struggling with, questions they may have, or anything else of their choosing. Several students used this time to make up their own story problems that they later exchanged with other students.

Presentation and Analysis of the Results

In order to assess the effectiveness of the interventions, students were given the same interview that had been given prior to the implementation of the interventions (Appendix A). Students also completed the same test (Appendix B and C) and were given the same survey (Appendix D).

Site A

After the interventions, the classroom teacher re-administered the student interviews. Again, just like the pre-interview, the students being interviewed were asked to arrange four numbered cards so that they would form a number as close to 5,000 as possible. The cards had the numbers five, nine, two, and six on them. After watching the students complete the task, the interviewer asked questions like, "What number did you make?" "Why did you put that number there?" and "How do you know that this number is the closest to 5,000?" The interviewer was encouraged to deviate from the prepared questions into other areas that would delve further into the child's ability to communicate. As on the pre-intervention interview, the children were able to get the correct answer rather quickly. The interviewer recorded this result consistently. The children proved to be strong in this area.

When asking students some deeper questions about how and why they got their answer, this time, the interviewer recorded much more thoughtful answers. For the most part, the children were able to communicate much more successfully. One child was asked the question, "How did you decide to put the nine where you did?" He responded in a way that was very typical of the post-interview, saying that, "the nine was the biggest number so I thought that should go in the biggest column." The interviewer responded, "Which column was that?" The child answered confidently, "The thousands column." This dialogue was indicative of many of the interview answers. They showed growth in several areas such as confidence in using mathematical vocabulary, a general comfort with communicating orally, and a better understanding of their thought processes. There certainly were a few times that the students' answers demonstrated quantity over quality. In these cases, the amount of communication was the only difference noted. While this is an important difference and evidence of some growth, these students still have a lot of room to grow before they can become successful mathematical communicators. However, at least they seemed to feel more comfortable with communicating mathematically.

The posttest for Site A, given in May, was titled "Dream Christmas" (Appendix B). As with the pretest the children were given a catalog showing items available for purchase at "The Twenty Dollar Store." They were assigned to purchase Christmas presents for family and friends at this unique store that only accepts twenty-dollar bills as payment. Then students were asked to calculate how many twenty-dollar bills they would need to give the cashier in order to cover the cost of their purchases. Finally, students were asked to calculate the change that they would receive. Students would be

graded on how well they explained what the answers were, how they arrived at those answers, and why they did what they did to get their answer. They were required to explain, in detail, everything they did to complete the tasks mentioned above.

The researchers developed a rubric for this written posttest that would assess students' abilities to communicate what the answer was, how they got that answer, and why they chose to do what they did. The rubric is based on the rubric used by the Illinois Goals Assessment Program for scoring their mathematical written communication problems. Their rubric was adjusted to delve deeper into what makes good mathematical communication and in order to make it more specific to this situation. The children would earn points based on their abilities to communicate the what, how, and why in the three following tasks: a) Finding the total cost, b) Finding how many twenty dollar bills were needed to pay for the items, and c) Finding how much change they would receive back. Students earned 3 points for communicating what their answer was in all three tasks, 2 points for communicating what their answer was in two of the tasks, 1 point for communicating what their answer was in one of the tasks and 0 points if they communicated what they did in none of the areas. The overall perfect score was 9, if they communicated in all areas successfully, while the minimum was 0 for not communicating their results in any of the areas mentioned. It should be noted that whether or not the answer given was correct was of no factor in this research. Only their effort to communicate what they did was assessed, even if the wrong answers were given.

This posttest was completed in May 1999. The comparison scores are outlined in Figure 17, Figure 18, Figure 19, and Figure 20.

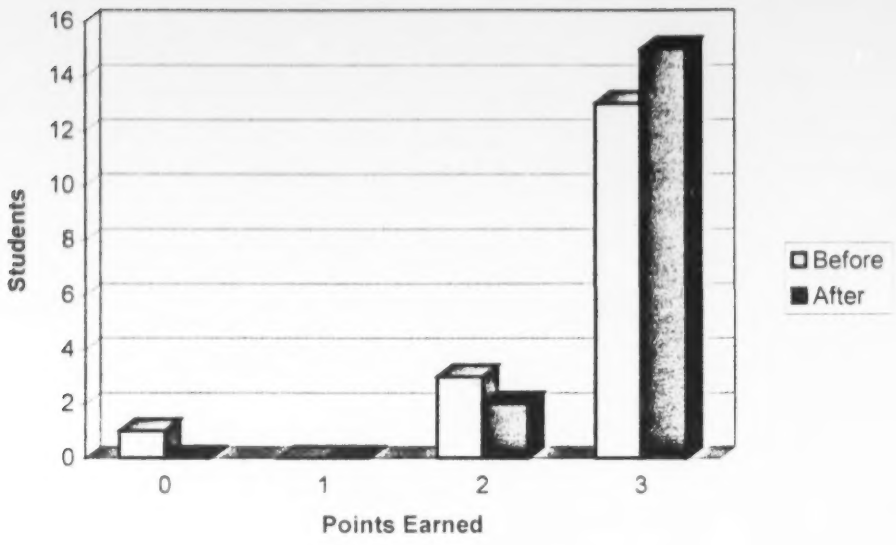

Figure 17. Student scores on pretest compared to the posttest for the "what" component at Site A.

Of the seventeen students tested, one earned a 0 for the "what" component of the pretest compared to none on the posttest. None of the students scored 1 on either the pre or posttest. Three of the students earned a score of 2 on the pretest compared to two on the posttest. A full thirteen out of seventeen scored a perfect score of 3 on this portion of the pretest, rising to fifteen out of the seventeen students on the posttest. On the pretest, 76% of the students in this class were able to successfully communicate what their answers were to each part, while only 6% scored less than 2. On the posttest, 100% of the children scored at least a 2 on the posttest, with a full 88% scoring a perfect score of 3 on the "what" portion of the posttest. While the pretest results clearly show that communicating the answer is not a problem at all for the students at Site A, they still showed improvement on this portion of the test, also leaving very little room to get even

better at this aspect of communication in mathematics. The intervention appears to have had a positive effect on the mathematical communication skills of the students at Site A.

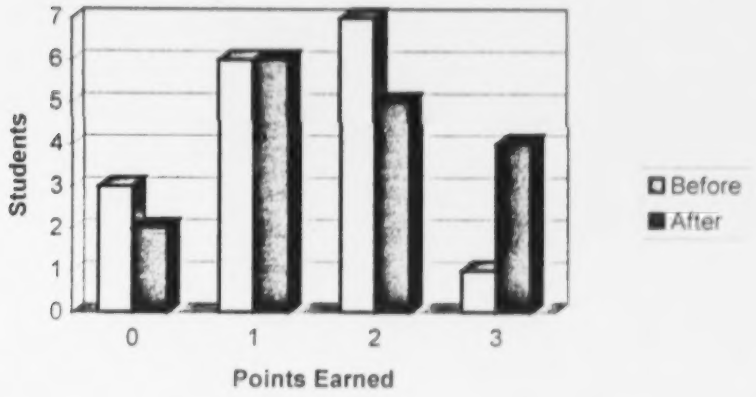

Figure 18. Student scores on pretest compared to the posttest for the "how" component at Site A.

Of the seventeen students tested, three earned a 0 for the "how" component of the pretest decreasing to only two students on the posttest. Six of the students scored 1 on both the pre and posttest. Seven of the students earned a score of 2 on the pretest compared to only 5 on the posttest. Only one child out of seventeen scored a perfect score of 3 on this portion of the pretest, while that number climbed to 4 on the posttest. Before interventions, 53% of the students in this class were only able to successfully communicate how they arrived at their answers on none or one task on this pretest. A similar sized group of 47% were able to successfully communicate how they arrived at their answers on two or three of the tasks. Only 6% received a perfect score. After the interventions, 47% of the students in this class were able to successfully communicate how they arrived at their answers on none or one task on this posttest. This indicates a

decline of 6% of students scoring in what would be considered the unsatisfactory range. Fifty-three percent were able to successfully communicate how they arrived at their answers on two or three of the tasks. The biggest gains after the interventions were seen in the amount of children earning a perfect score. That percentage climbed 17%. While before the interventions, the majority of the children clearly fell into the category of successful communication on one or two tasks, after the interventions, the majority of children fell into the category of successful communication of how they got their answers, scoring a 2 or a 3. Again, the interventions appear to have been successful at improving the mathematical communication of students when related to sharing how they arrived at their answers.

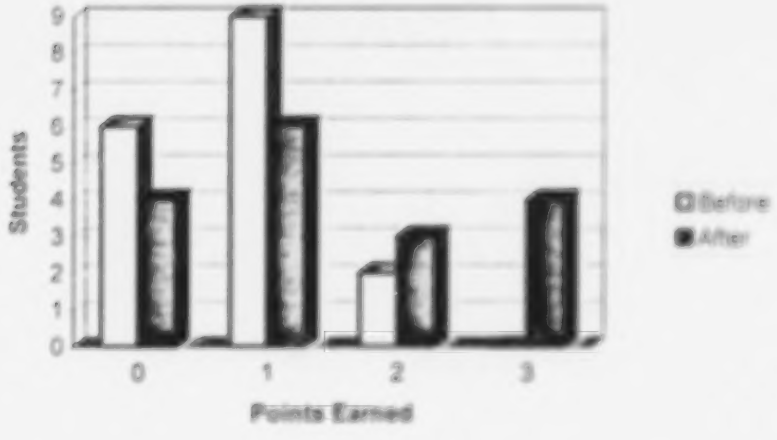

Figure 19. Student scores on pretest compared to the posttest for the "why" component at Site A.

Of the seventeen students tested, six earned a 0 for the "why" component of the pretest and nine of them scored 1. This is compared to four scoring a 0 and six scoring a 1 on the posttest. While two of the students earned a score of 2 on the pretest, three

received the same score on the posttest. No children out of seventeen scored a perfect score of 3 on this portion of the pretest, while four students were able to earn a perfect score of 3 on the posttest. A full 88% of the students in this class were not able to successfully communicate why they chose to do what they did, scoring 1 or lower, before the interventions. After the interventions, that percentage declined to 59%, almost a 30% drop-off. On the pretest, only 12% were able to successfully communicate why they did what they did, scoring 2 or 3, while on the posttest, 41% were able to successfully communicate why they did what they did. This is a drastic jump of 29%. The majority of the children were unable to successfully communicate before the interventions. This improves after the interventions with solid movement evident in the students' abilities to communicate why they chose to do what they did when solving a problem.

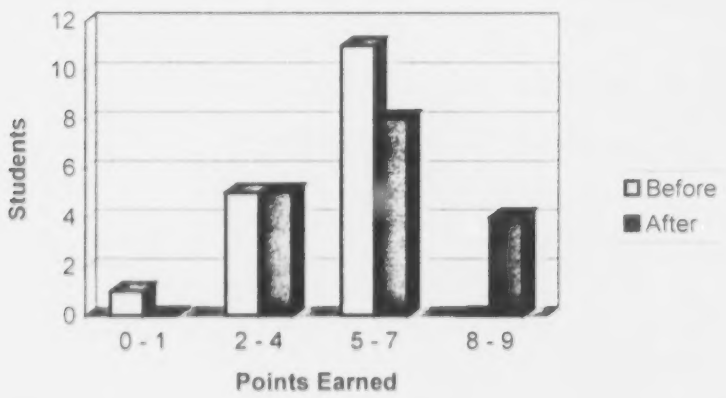

Figure 20. Student overall scores on pretest compared to the posttest at Site A.

. Of the seventeen students tested, none of them were able to score an 8 or a 9 overall on the pretest, while four were able to do so on the posttest. Eleven scored in the 5 to 7 range, before interventions, compared to eight after the interventions. Three of those students moved into the highest scoring range. On both the pre and posttest, five of them

earned a score in the 2 to 4 range. While one child out of seventeen scored in the 0 to 1 range before interventions, none scored in the 0 to 1 range after the interventions. Not a single child was able to communicate in all areas successfully before the interventions. This jumped to 24% who could communicate in all areas successfully after the interventions. While the overall results did not show significant changes in students' scores in all areas, an increase was clearly evident for those students who moved into the highest range. The average overall score for students on the posttest was 5.9, increasing from 4.8 on the pretest. This coupled with the fact that a full eleven out of seventeen students, or 65%, improved on their individual score from the pretest to the posttest, clearly proves that the intervention was successful.

A survey was another tool that was used to collect data about student perception of their abilities as math students. The survey also asked students to rate how often they are asked to explain how they found their answers to math problems out loud or in writing.

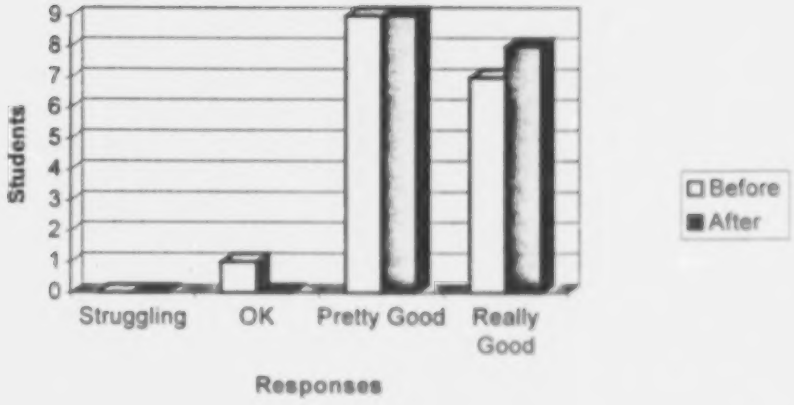

Figure 21. Student responses from Site A recorded before the interventions compared to those recorded after the interventions to the question, "Which of these would best describe you as a math student?"

When given a survey before the interventions, the students' responses at Site A demonstrated a confidence in their math abilities. No student described himself as a struggling math student, and only one said they were just OK at math. However, nine students said that they were pretty good at math, while seven responded that they were really good at math. Thus, 94% of the children have a positive view of their math abilities compared to only 6% who recognize that math may be a weakness for them. These results changed only slightly after the interventions. After the interventions there were not any children who considered themselves struggling or even just OK math students. Nine students considered themselves to be pretty good math students, while eight considered themselves to be very good math students. In comparison, one more child considered himself very good at math on the posttest and one less considered

himself as an OK math student. The changes evident in student's perception of their math abilities are negligible.

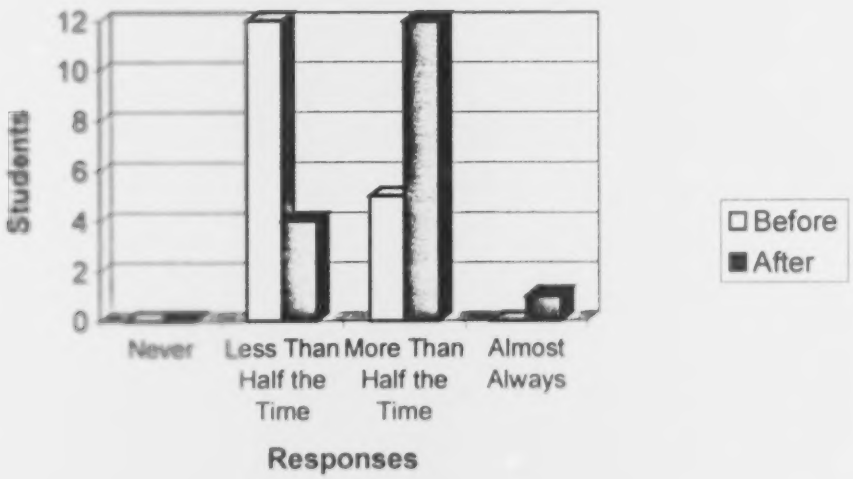

Figure 22. Student responses from Site A recorded before the interventions compared to those recorded after the interventions to the question, "How often are you asked to explain in writing the way you found an answer to a math problem?"

Before and after interventions, there were no students who thought they were never asked to explain in writing their method for answering a math problem. Twelve students thought that this was asked of them less than half the time before the interventions. This number decreased to four after the interventions. Another five of the children thought that they had been asked more than half of the time to explain in writing how they solved a problem in the beginning, while in the end, twelve students felt this way. No student thought that he was always asked to describe in writing how he solved a math problem before the interventions. After the interventions, one child thought that he was always asked to describe in writing how he solved a math problem. These data indicate that part of students' problems with mathematical communication can be traced

to this lack of experience with communicating in written form. Before the interventions a full 71% of students thought they were asked to communicate their methods and thought processes less than half the time. This percentage decreased to 24% after the intervention. By contrast, the percentage of students who thought they were asked to explain mathematical answers in writing more than half the time increased from 29% to 71%. This is a large increase and can, perhaps, be correlated with the increases in the posttest scores of the students. It appears that the more experiences the students are given in communicating their answers in written form, the better they will be at doing it.

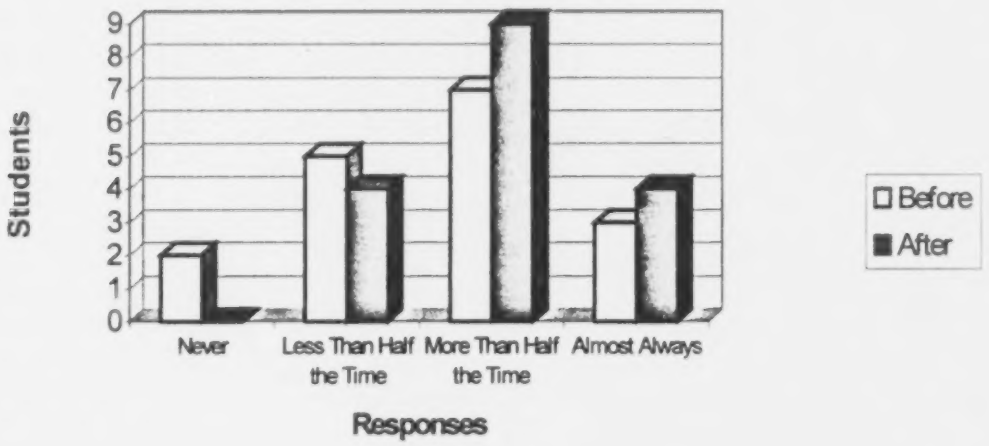

Figure 23. Student responses from Site A recorded before the interventions compared to those recorded after the interventions to the question, "How often are you asked to explain aloud the way you found an answer to a math problem?"

Before the interventions, in response to this question of the survey, two students responded that they were never asked to explain aloud the way they found an answer to a problem. This decreased to no students after the interventions. In the beginning, five

students thought that they were asked to explain aloud their methods for answering a problem less than half the time. This number decreased to 4 after the interventions. Seven students responded before the interventions that they were asked to explain aloud the way they solved a math problem more than half the time, increasing to eight after the interventions. Before the interventions, three students indicated that they were always asked to explain their answers and methods aloud before the interventions, while four responded that way after the interventions. While the numbers for this question before the interventions definitely reflect a stronger emphasis that educators put on oral communication over written communication, they still leave 41% of students thinking they are asked to communicate in this way less than half the time. In contrast, after the interventions, 76% of students thought that they were asked to explain orally their answers to math problems either more than half the time or all the time. These data from before the interventions further suggest the students' lack of experience in communicating mathematically before the intervention, specifically in oral form. While the clear increase in students' experiences in explaining answers out loud might be considered as one of the reasons for the increase in the students' posttest scores.

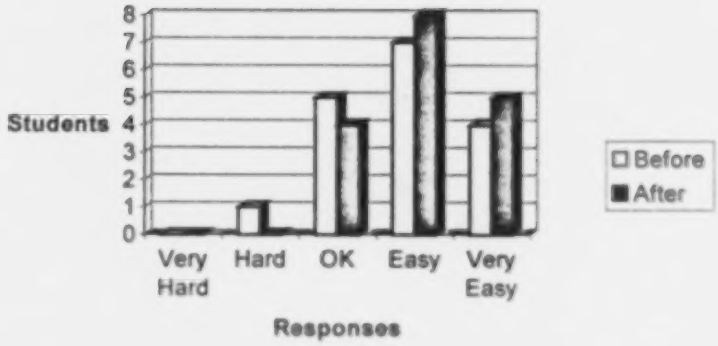

Figure 24. Student responses from Site A recorded before the interventions compared to those recorded after the interventions to the question, "How easy is it for you to explain how you solved a math problem?"

Before the interventions, four out of seventeen students considered it to be very easy for them to explain how they solved a math problem. This increased to five students after the interventions had been completed. Also before the interventions, out of seventeen, seven responded that it was easy to explain how they solved a math problem. Again, this increased by one after the interventions, to eight. Five out of seventeen students felt they were just ok at such explanations before the interventions. This decreased to four out of seventeen after the interventions. Only one thought that such explanations were hard before the interventions, which dropped to none after the interventions. No one thought that explaining how they solved a math problem was very hard. Neither before nor after the interventions did any students think that explaining how they solved a math problem was either hard or very hard. This graph shows an increase of 11% in how many students find it to be at least easy to explain how they answered a math problem. Before the interventions, 35% lacked confidence, as indicated

by an OK or less response, in their ability to communicate their solutions, compared to 24% after the interventions. Some increase in the students' confidence in explaining their answers is evident after the interventions. The intervention again appears to have had a positive effect on students' confidence levels.

Site B

After the interventions the classroom teacher re-administered the student interviews. Again the students were asked to arrange four numbered cards into a number as close to 5000 as possible. The four numbers were five, nine, two and six. Most of the students were successful in the task. When asked to explain their answer the students were not as successful. However, they were able to explain with a little more clarity than before the interventions were implemented. For example: one student was asked why she made the number 5,269. Her response was, "the two is closer to zero than the six or the nine. If it were a zero, it would be even closer to 5,000." Another student answered, "The five is in the thousands. The two is the smallest, six is the second lowest, and nine is the highest." Yet another student explained that, "200 is only 200 off of 500. That's why I put the two in the hundreds place."

The posttest for Site B consisted of one multiple-part open-ended question involving earning money by scheduling tasks for one week. It was identical to the pretest. The main purpose for this test was to evaluate how well the students communicated their answers. The rubric developed by the researchers has three parts (Appendix D). These three parts are "what," "how," and "why." Students were evaluated on if they told what the answer was in their written explanation. Students were evaluated on if they explained how they came up with their answers. Students were also

evaluated on if they explained why they did their work the way they did. Each portion of the rubric earned a score from 0 to 3. A score of 3 was given if a student told all parts of what the answer was. A score of 2 was given if a student told most parts of what the answer was, and a score of 1 was given if the student gave less than half of what the answer was in his written explanation. A score of 0 was given if students did not tell any part of what the answer was in their written explanation. The same standards were applied to the other two parts of the evaluation.

This posttest was completed in May 1999. Comparison results of the pretest and the posttest are outlined in Figure 25, Figure 26, Figure 27, and Figure 28.

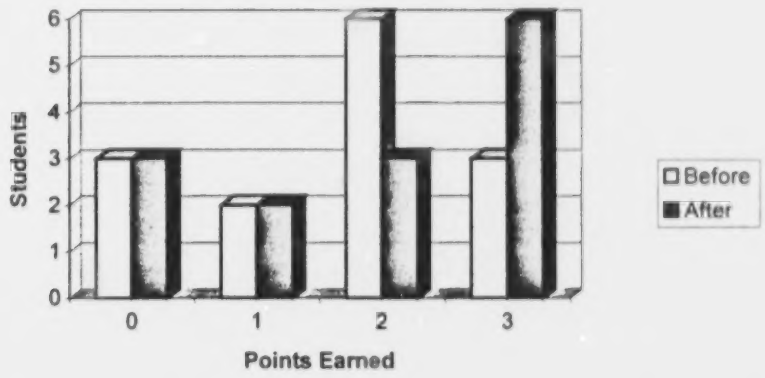

Figure 25. Student scores on the "what" portion of the pretest given at Site B compared to the posttest.

Of the fourteen students tested, three earned a score of 0 on the "what" portion of the pretest. Two students received a score of 1, six received a score of 2, and three received a score of 3. Sixty-four percent of the students scored either a 2 or a 3 on this portion of the rubric. Most of the students told most or all of what the answer was in their written response.

On the posttest three students earned a score of 0 on the "what" portion of the posttest. Two students received a score of 1, three received a score of 2, and six received a score of 3. Sixty-four percent of the students scored either a 2 or a 3 on this portion of the rubric. Most of the students told most or all of what the answer was in their written response. The only change that occurred in this portion of the test was the increase of students who gave complete answers. Twice as many students earned a 3 on the "what" portion of the rubric.

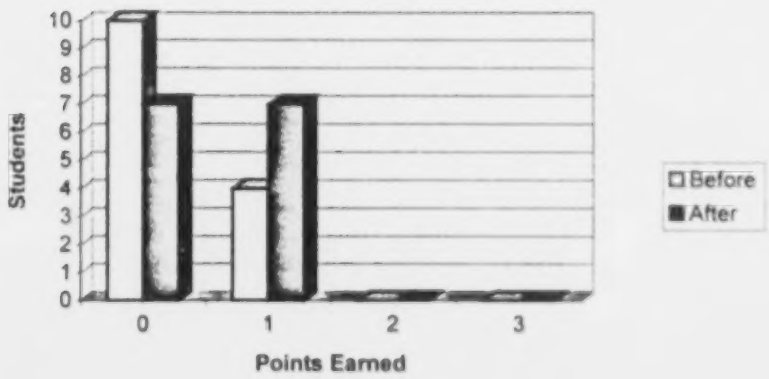

Figure 26. Student scores for the "how" portion of the pretest given at Site B compared to the posttest.

Of the fourteen students tested, ten received a score of 0 on the "how" portion of the pretest. These students did not offer any explanation of how they arrived at their answers to the question posed. Four of the students earned a score of 1. These students explained how they found the answer to less than half of the question. None of the students scored a 2 or a 3 on this portion of the pretest.

On the posttest seven students received a score of 0 on the "how" portion. Seven students received a score of 1. There was some improvement in the "how" portion of the students' written explanations. Most of the students were successful in telling what the answer was, but none of them explained fully how they arrived at their answers.

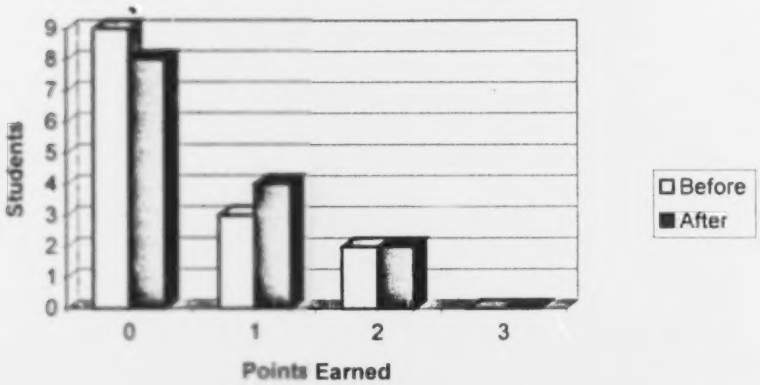

Figure 27. Student scores for the "why" portion of the pretest given at Site B compared to the posttest.

Of the fourteen students tested, nine received a score of 0 on the "why" portion of the pretest. Three students received a score of 1. Two students received a score of 2, and zero students received a score of 3 on the "why" portion of the pretest. Sixty-four percent of the students had no explanation at all about why they did the things they did in their problem. Only 22% of the students gave minimal explanation of why they did the things they did, and 14% of the students explained more than half of why they did things the way they did them. Eighty-six percent of the students, a large majority, told little or nothing about why they did their problem in the manner that they did it.

On the posttest eight students received a score of 0 on the "why" portion of the pretest. Four students received a score of 1. Two students received a score of 2, and zero students received a score of 3 on the "why" portion of the posttest. There was minimal improvement in this area as only one student improved in his explanation of why he did what he did to solve the problem.

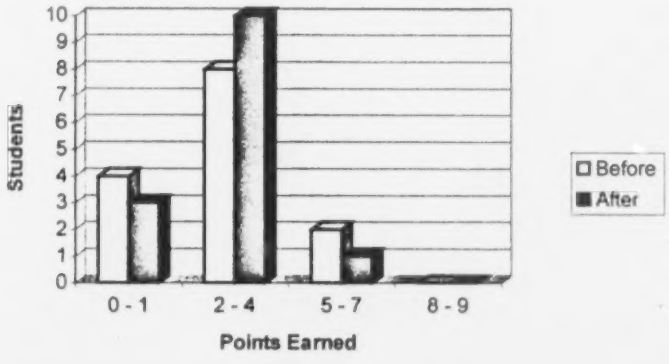

Figure 28. Student overall scores on pretest at Site B compared to the posttest.

Four students received an overall score of 0 or 1 on the pretest. Eight students received a score of 2, 3, or 4, and two students scored between 5 and 7. Eighty-six percent of the students scored less than 5 points on the pretest, while 14% scored 5 or higher.

On the posttest three students received an overall score of 0 or 1. Ten students received a score of 2, 3, or 4, and one student scored between 5 and 7. There was not a noticeable improvement in the overall student scores. There was a slight rise in the average student score.

A survey was another tool that was used to collect data about student perception of their abilities as math students. The survey also asked students to rate how often they

are asked to explain how they found their answers to math problems out loud or in writing.

Figure 29. Student responses to the question, "Which of these would best describe you as a math student?" from Site B before and after the interventions.

At Site B no students responded that they were struggling math students before the interventions. One student answered that he was an okay math student. Eight students felt that they were pretty good math students, and five responded that they were really good math students. Ninety-three percent of the students surveyed said that they were either pretty good or really good as math students. This shows that most of the students at Site B were confident in their abilities as math students.

On the post-survey no students responded that they were struggling math students. Three students answered that they were okay math students. Six students felt that they were pretty good math students, and five responded that they were really good math students. Again the students showed a confidence in their math abilities.

The students did not evaluate their performance on the written portion of the posttest in their assessment of their abilities as math students. Even though the students

did not do well in the testing they still felt good about their mathematical abilities. This shows a discrepancy between performance and self-perception.

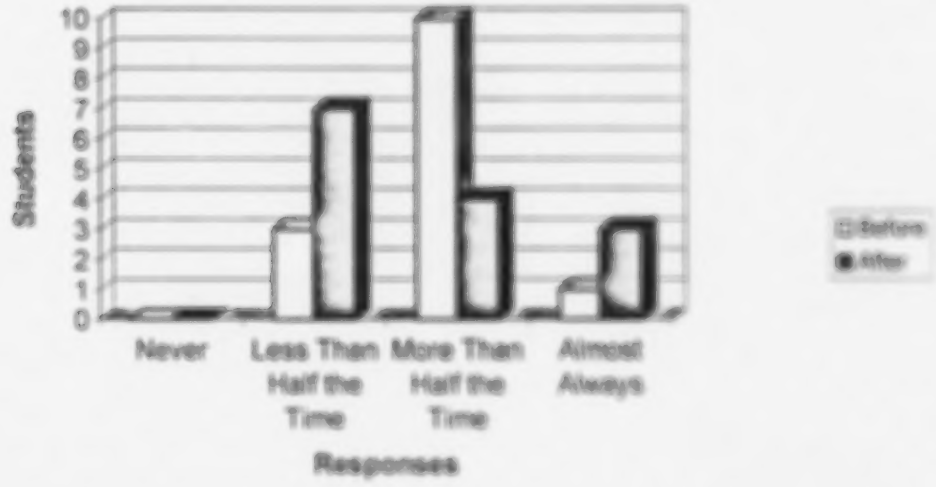

Figure 30. Student responses to the question, "How often are you asked to explain in writing the way you found an answer to a math problem?" from Site B before and after the interventions.

At Site B no students responded that they were never asked to explain in writing how they solved a math problem before the interventions. Three students answered that they were asked less than half the time to explain in writing how they solved a math problem. Ten students felt that they were asked to explain in writing how they solved a math problem more than half the time, and one responded that they were almost always asked to explain in writing how they solved a math problem. A large majority of the students, 79%, responded that they were asked to write about math most or almost all of the time. This shows that the students' perception was that they were writing a significant portion of the time during math.

After the interventions at Site B, no students responded that they were never asked to explain in writing how they solved a math problem. Seven students answered that they were asked less than half the time to explain in writing how they solved a math problem. Four students felt that they were asked to explain in writing how they solved a math problem more than half the time, and three responded that they were almost always asked to explain in writing how they solved a math problem.

Student perception was that they were asked to write about their math less after the intervention than they were asked to write before the intervention. It could be that students became more aware of what it meant to write about math. Before the intervention the students were less conscious of what it meant to write about their math thinking.

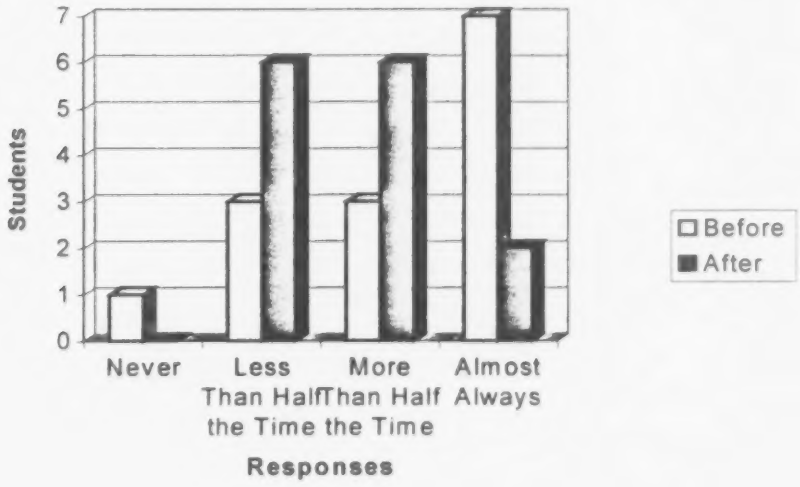

Figure 31. Student responses to the question, "How often are you asked to explain aloud the way you found an answer to a math problem?" from Site B before and after the interventions.

At Site B one student responded that he was never asked to explain aloud how he solved a math problem before the interventions. Three students answered that they were asked less than half the time to explain aloud how they solved a math problem. Three students felt that they were asked to explain aloud how they solved a math problem more than half the time, and seven responded that they were almost always asked to explain aloud how they solved a math problem. Half of the students responded that they were asked to explain how they solved math problems out loud almost always.

After the interventions no students responded that they were never asked to explain aloud how they solved a math problem. Six students answered that they were asked less than half the time to explain aloud how they solved a math problem. Six students felt that they were asked to explain aloud how they solved a math problem more than half the time, and two responded that they were almost always asked to explain aloud how they solved a math problem.

Again the student perception of the frequency of responses decreased. Overall students felt that they were asked to respond out loud less often during the intervention period than before. It could be that students became more aware of what it meant to talk about math. Before the intervention the students were less conscious of what it meant to talk about their math thinking. After the interventions they were keenly aware of when they did it, and, conversely, when they were not doing it.

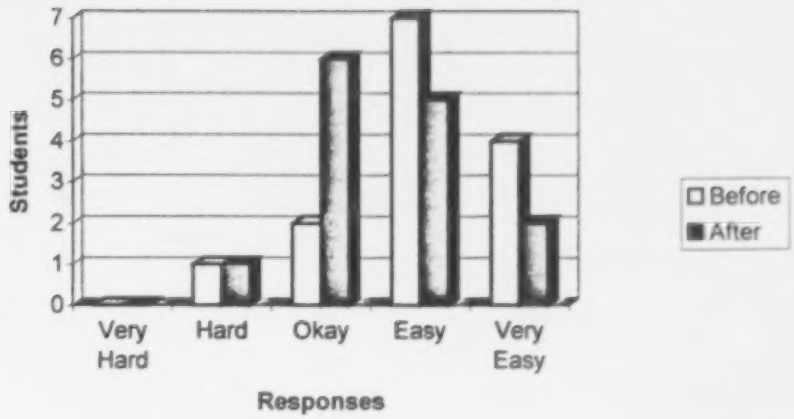

Figure 32. Student responses to the question, "How easy is it for you to explain how you solved a math problem?" from Site B before and after the interventions.

At Site B no students responded that explaining a math problem was very hard for them before the interventions. One student responded that explaining math problems was hard for him. Seven students felt that it was easy for them to explain how they solved a math problem, and four students said that it was very easy to explain how they solved a math problem. A majority of the students, 79%, responded that they felt that it was easy or very easy for them to explain how they solved math problems before the interventions.

After the interventions no student responded that explaining a math problem was very hard to do. One student thought that it was hard. Six felt okay at it. Five responded that it was easy and two that it was very easy to explain how they solved a math problem.

The students' perceptions of their ability to explain how they did a mathematical process went down after the interventions. The purpose of the intervention was to increase the students' ability to explain their mathematical thinking yet their confidence in this ability decreased. One possibility is that the children became more aware of what it means to explain their ideas to others. Before the intervention the students were not as

familiar with explaining their ideas in math. During the course of the interventions these children found out what it really means to have to explain themselves and their mathematical ideas.

Students realized that they were not as good as they thought at communicating mathematically. Even with this realization they did not see their math abilities as lacking. Conceptually, the students did not connect communicating their math ideas with their ability to do math. This is how math has been treated in the past, so it is not surprising that students would miss the connection between math and communication as well. A student considers himself good at math, just not good at explaining it.

Conclusions and Recommendations

Based on the presentation and analysis of the data from the pre and posttests, the students at Site B did not show a significant improvement in their ability to communicate mathematically, while the students at site A did show fairly significant improvement in their abilities to communicate mathematically. The students at Site B did show significant increase in their ability to communicate what their answers were. However, this was the only area of the posttest in which students at Site B showed any significant improvement. While, the students at Site A showed their most significant improvements in the "how" and "why" portions of the posttest.

Based on these results, the researchers conclude that the skills presented in the interventions were more age appropriate for the fourth grade students at Site A. Without some form of concrete help such as the use of manipulatives, the third grade students at Site B were only ready to comprehend the idea of communicating what their answer was. The "how" and "why" of communicating their answers proved to be too difficult for the

students at this level when only using more abstract interventions such as the ones used in this research. By contrast, the students at Site A were already fairly proficient in communicating what their answers are and intellectually ready to move on to explaining how they got their answers and why they did what they did to get to their answers. The interventions successfully moved students at Site A from being proficient at communicating only the "what" of their answers towards being proficient at communicating the "how" and "why" of their answers. These same interventions were only able to improve the students at Site B in the area of explaining what their answer is.

These researchers believe that this lack of improvement at Site B was due to the use of interventions that were more suited for students at a 4th grade or higher level. Because the students at Site B were in the third grade, there were certain cognitive limitations that can not be overcome through the use of abstract interventions. It is believed that these students would have benefited from the use of manipulatives and, in general, more concrete interventions that would allow them to see the what, how, and why of solving their problems. In addition to this, younger students need more time to learn and process concepts. Therefor, a longer period of intervention was needed.

It is recommended to researchers who might want to duplicate this research that they include a formal survey of teacher at the research site. The survey would ask teachers from earlier grades about the frequency that they have required students to communicate in both oral and written form during math time. This would give further proof as to the reasons behind the problem of students' inability to communicate mathematically. This information could be compared with the data from the student survey, allowing comparison of perception of students as to how often they are asked to

communicate mathematically and perception of teachers as to how often they ask students to communicate in math class. These researchers believe that this might provide valuable insight into the reasons such a problem exists and also might give more accurate feedback about the frequency of students' experiences communicating in math before interventions have been implemented.

In regards to future interventions at Site A and B, the researchers recommend that the selected interventions should continue with a few modifications. At Site A, in the fourth grade classroom, the cooperative group problem solving with real life problems will continue as it was implemented during the intervention period. It is recommended that the journal writing be increased to become a daily occurrence. This will communicate to students the importance of writing in a math program. This increase in writing will also show that writing is an integral part of math that is a requirement not just a special assignment or a fringe activity. Finally, the researchers would add another component to the math program at Site A. This component would include activities that would emphasize building math vocabulary. This added intervention should complement all the other interventions by giving students the words to be a better communicator. Math vocabulary would be discussed for each unit and students would be held accountable through math vocabulary quizzes to check their knowledge of the words meaning and through analysis of their journal writing to see if they are using the math vocabulary words properly.

At Site B, the selected interventions will be adjusted to better accommodate the needs of the third grade students. The cooperative group problem solving with real life problems will continue with a greater emphasis placed on solving more hands-on

problems that require the use of manipulatives to solve them. This will help students develop a better understanding of the processes they used and give them something more concrete to talk about. This is an important piece to add as their journal entries during the interventions indicated that they had trouble with communicating more abstract concepts. The concrete manipulatives will be more appropriate for their grade level and thus will produce better results, especially in the areas of explaining how they solved the problems and why they did what they did. These were the areas that the students at Site B showed the least improvement. Additionally, these researchers suggest a longer intervention period is needed with such young students. As with Site A, the journal entries would be increased to occur daily. Also like with Site A, there will be an added component of developing math vocabulary to give students the words they need to be better communicators.

It is anticipated that these changes will produce even better results than those achieved during the intervention period. The changes should allow students at Site B to move beyond just improving in communicating what their answers are and allow them to have more insight into how and why they did what they did. The students at Site A should benefit greatly from increasing their math vocabulary. The researchers anticipate that this will increase the growth that was already evident after the intervention period. These fourth grade students will become even more effective communicators, both orally and in written form, once they have more effective and efficient words to use to describe what they are thinking. The researchers strongly recommend these interventions along with the modifications outlined above to teachers looking to increase their students' abilities to communicate mathematically.

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Appendices

APPENDIX A

STUDENT INTERVIEWS

INTERVIEW

Activity

- Given four number cards (5,9,2,6) student will make a number as close to 5000 as possible.

Questions (Interviewer may use these questions as a starting point.)

- What number did you make?
- Tell me why you made that number.
- Why did you put that number there?
- How do you know that that number is the closest to 5000?
- How close is your number to 5000?

APPENDIX B

PRE AND POSTTEST ADMINISTERED AT SITE A

DREAM CHRISTMAS

It is a week before Christmas and you are going shopping to get people presents. You can spend any amount of money that you want, but the catch is, you have to shop at **The 20 Dollar Bill Store**, and they only accept 20 dollar bills. Go shopping for the items listed on their advertisement. Choose as many as you want. List the items you choose and their cost on the chart below along with who they are for. Then decide how many $20 bills you will need to give the cashier in order to pay for them and how much change they will have to give you. Explain all your work on the lines at the bottom.

THE LIST:

WHO	WHAT	HOW MUCH

EXPLAIN EVERYTHING YOU DID:

The Dollar Bill Store

Gardening Gloves and tool = $7.75

Stapler = $5.50

Trumpet = $125.00

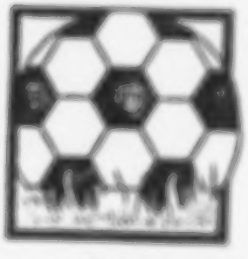
Violin $75.95

Fan = $10.45

Teddy Bear = $16.00

Fake nose and glasses = $2.75

New Tennis Shoes = $45.00



Soccer Ball = $25.00

Plant = $8.95

Tea Kettle = $27.00

Christmas Ornament = $11.50

Eye Glasses = $47.00

Toy Block = $4.50

Briefcase = $36.00

Binoculars = $70.00

APPENDIX C

PRE AND POSTTEST FOR SITE B

Third grade math item.

On Monday, Joe asked his mom if he could go to the movies on Friday. His mother gave him a list of chores and said he would have to earn the $5.00 to buy the movie ticket.

Making his bed: 2 quarters	Washing dishes: 2 quarters
Cleaning his room: 1 quarter	Taking out trash: 1 quarter
Sweeping the floor: 2 dimes	Folding clothes: 2 dimes

Make a plan for Joe to earn the $5.00. List the chores he will need to do each day. Explain in words how you found your answer and why you did the plan the way you did.

APPENDIX D

STUDENT SURVEYS

Name: Date:

Highlight your answer.

Which of these would best describe you as a math student?

Struggling **OK** **Pretty good** **Really good**

How often are you asked to explain in writing the way found an answer to a math problem?

Never **Less than half the time**

More than half the time **Almost always**

How often are you asked to explain aloud the way found an answer to a math problem?

Never **Less than half the time**

More than half the time **Almost always**

How easy is it for you to **explain** how you solved a math problem?

Very Hard **Hard** **OK** **Easy** **Very Easy**

APPENDIX E

SAMPLE PROBLEM SOLVING ACTIVITIES

ACT OUT OR USE OBJECTS Name

15 Donna was putting six new bears in the display case at the toy store. The case had three shelves, one on top of the other, with two spaces on each shelf. Each bear had a name: Abby, Bobby, Cathy, Dorothy, Eric, and Forrest. Donna put Dorothy next to Eric and above Forrest. She did not put Bobby next to Eric or Forrest. She did not put Abby next to Bobby. Where did Donna put each of the bears?

FIND OUT

- What is the question you have to answer?
- How many bears are there?
- What do you know about where Donna put Dorothy?
- What do you know about where Donna put Bobby?
- What do you know about where Donna put Abby?

CHOOSE A STRATEGY

- Circle to show what you choose.

SOLVE IT

- If you use a piece of paper for each bear, how many pieces of paper do you need? How are you going to mark the papers?
- Where can you put the paper for Dorothy?
- If you put Dorothy in one place, then can you find a place for Eric? What about Forrest?
- What do you know about where Bobby goes? Is there only one place that is not next to Forrest or Eric?
- Where does Abby go?
- Is there only one space left? Which bear is left?
- Is there another way to arrange the pieces of paper with the same directions?

Dorothy	

LOOK BACK

- Read the problem again. Look at the information given and the main question. Review your work. Is your answer reasonable?

MAKE AN ORGANIZED LIST

Name

11 Meredith got on the bus and dropped her coins into the money meter. Her bus fare was 15 cents. How many different groups of coins could Meredith have used to pay for her bus fare?

FIND OUT

- What is the question you have to answer?
- What did Meredith do? How much was her bus fare?
- What kinds of coins could she have used?
- How much money would be in each group of coins?
- Would one dime and five pennies be a different group than one dime and one nickel?

CHOOSE A STRATEGY

- Circle to show what you choose.

SOLVE IT

- How many columns are there in the list started on your paper? What is at the top of the first column? What is at the top of the second column? What is at the top of the third column?
- Look at group 1. What is the greatest number of dimes Meredith could use? How many nickels could she use with the dime? What is the sum of one dime and one nickel? Would she need to use any pennies with the dime and nickel? Notice the zero in the pennies column.
- Look at group 2. Could Meredith use a dime again but have a different group of coins?
- Look at group 3. Could Meredith use a dime again and have a different group of coins? What is the greatest number of nickels she could use? How many pennies would she use with the nickels?
- Finish the list on a separate recording sheet. How many different groups of coins could Meredith have used to pay her bus fare?

	Dimes 10¢	Nickels 5¢	Pennies 1¢
Group 1	1	1	0
Group 2	1	0	5
Group 3			

LOOK BACK

- Look back to see if your answer fits with what the problem tells you and asks you to find. Read the problem again. Look back over your work. Does your answer fit?

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